

Quantitative approach into the heat transfer by convection in microelectronics

by B. Więcek

Institute of Electronics, Technical University of Łódź, 18-22 Stefanowskiego str. 90-924 Łódź, Poland

Abstract

Heat is dissipated to the ambient by convection and radiation in every electronic device. In the long chain of thermal conductors from a heat source (semiconductor) to the ambient (fluid), convection has its significant contribution, especially in steady, long-term conditions. This paper presents 2-D modelling of convective heat dissipation for vertically placed electronic elements with non-uniform temperature distribution. One describes heat removal by convection using the convective transfer coefficient α , which depends upon the temperature and the component size. In this paper we evaluate the transfer coefficient by 2-D natural convection simulation.

1. Introduction

Recent publications indicate a growing interest of thermal analysis in semiconductor power devices and integrated circuits. In the majority of the published works, convection is described by approximated transfer coefficients taken from tables. Many authors assume a uniform distribution of the transfer coefficient, even for large heat removing surfaces [1-2].

In practice, heat transfer by convection is very important in any thermal modelling of microelectronic devices, and has an influence on the accuracy of the entire heat removal model. Although the problem of convection is well known in non microelectronic applications [1-3], it is still not included into the compact thermal analysis, because of high difficulties in getting satisfactory results. There are known analytical [1-2], simplified 1-D [1-2,4] and a 2-D numerical solutions of the Navier-Stokes (*N-S*) equations [3]. This paper describes 2-D numerical approach of convection modelling for vertical electronic components with a highly non-uniform distribution of the temperature. The main goal of this work is to confirm or update the frequently proposed non-linear characteristics of the convective transfer coefficient derived for the constant temperature of the heat source, and by solving the 1-D, simplified *N-S* equations ($\frac{1}{4}$ Law), [1-2,4].

$$\alpha_c = \alpha_0 \left(\frac{\Delta T}{\Delta T_0} \right)^{\frac{1}{4}} \quad (1)$$

where α_0 denote the transfer coefficient corresponding to temperature difference $\Delta T_0 = T - T_a$

A long resistor placed in the middle of the ceramic substrate (Fig.1) has been chosen for simulation and experiment. It provides a non-uniform temperature distribution, and measurable difference of temperature on the bottom and top side of the substrate caused by convection.

2. 2-D model for natural convection

A heat source-hybrid resistor of a few cm length and of a few tens of μm thickness (h_r) is placed on the ceramic substrate as shown in Fig. 1. We assume 1-D heat conduction in the substrate because of $h_s/h_r \gg 1$ and $h_s/t_s \ll 1$, where h_r , h_s and t_s are the resistor height, height and thickness of the substrate. The hybrid resistor is placed horizontally in the middle of a vertical ceramic substrate. The length and the height of the resistor are the following: $h_r=50\mu\text{m}$, $L=30\text{mm}$. The 2-D model for heat and mass transfer in the surrounding air is established for an

extended region, two times larger than the ceramic substrate height, because of unknown boundary conditions for temperature and velocity on the top of the substrate.

One assumes laminar flow of the fluid, which is valid for $10^4 < Gr_f Pr_f < 10^9$ [2]. For a typical size of microelectronic components e.g.: $h_s=3\text{cm}$, for temperature difference of maximum 200°C , and for the following air parameters in temperature $T_a=20^\circ\text{C}$: viscosity $\nu=1.56 \times 10^{-5} \text{m}^2/\text{s}$, thermal conductivity $\lambda_a=0.026 \text{W/Km}$, density $\rho=1.2 \text{kg/m}^3$, $c_a=1 \text{kJ/kgK}$, we can get: $Gr_f=7.25 \times 10^7$, $Pr_f=0.7$, which indicates that in microelectronics, the laminar flow of the air takes place during natural convection.

For steady state 2-D conditions one may consider only one part of the *N-S* equations set, i.e: the equation for vertical direction only, which takes the form [1,3].

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g \beta (T - T_a) \tag{2}$$

where: u , v , β and g denote the horizontal and vertical components of the velocity, thermal expansion coefficient and the acceleration of the gravity, respectively.

In eqn. (2) the pressure gradient has been replaced by the apparent weight force per unit fluid volume. The second *N-S* equation for the horizontal direction is neglected. The continuity and energy equations have the typical forms:

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= a \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \text{ where } a = \frac{\lambda_a}{\rho c_a} \end{aligned} \tag{3}$$

The numerical model of heat conduction in the substrate is derived from the energy conservation law, and can be expressed using heat fluxes [5-6]. There are source and sourceless elements in the substrate. For this work, a heat source of one-grid size is assumed. There is no limit for the number of source elements in the model, and the heat source can be easily extended by any number of one-grid source elements. The energy equations for source and sourceless elements take a form:

$$S_s dy \frac{\partial \phi_s(y)}{\partial y} + 2 S_a \phi_a(y) = \begin{cases} P_z \\ 0 \end{cases} \tag{4}$$

where: $\phi_s = -\lambda_s \frac{\partial T}{\partial y} \Big|_{x=0}$ and $\phi_a = -\lambda_a \frac{\partial T}{\partial x} \Big|_{x=0}$ are the heat fluxes in the

substrate and in the ambient, S_s and S_a are the corresponding areas where fluxes cross through, P_z [Wm] denotes the power per length unit dissipated in the hybrid resistor [5].

Finally, the energy equation for heat conduction in the substrate ($x=0$) can be expressed as:

$$- dx \lambda_s \frac{\partial^2 T}{\partial y^2} - 2 \lambda_a \frac{\partial T}{\partial x} = \begin{cases} P_z \\ 0 \end{cases} \tag{5}$$

where λ_s denotes the substrate thermal conductivity and dx is the thickness ($dx=t_s$).

There are no well defined boundary conditions for both the temperature and the velocity on the top and right sides of the region of laminar fluid flow (Fig.1). The boundary conditions derived from 2-D numerical model for semi-infinite area are used [5]. The solution can be easily obtained by introducing other variables and solving the set of 1-D ordinary equations by the Runge-Kutta method. We assume the same power dissipated for both models with similarly varying temperature on the substrate [5].

3. Numerical results

The numerical model presented above has been implemented in C-code, using the *Gauss-Crout Elimination* for solving the heat conduction equations in the substrate, and *Gauss-Seidel Iteration* method for solving the *N-S* equations. Both procedures run in series, one after the another. The temperature on the substrate obtained by heat conduction model has been used for calculating the temperature and velocity distribution of the air, and so on. A grid of 50x150 has been chosen.

The properties of the ceramic substrate and of the air are assumed not to depend upon the temperature and are as follows: $\nu=1.56 \cdot 10^{-5} \text{m}^2/\text{s}$ $T_a=300\text{K}$, $\lambda_s=20\text{W/Km}$, $\lambda_a=0.02\text{W/Km}$, $h=6\text{cm}$, $w=2\text{cm}$, $t_s=0.4\text{mm}$, $h_s=3\text{cm}$. As it was expected the temperature along the substrate and in the surrounding air is highly non-uniform (Fig.2). The vertical component of the fluid velocity has a maximum close to the substrate. This maximum shifts away from the substrate while the warm air moves up to the top of the heat source. Above the substrate, the temperature of the air drops down, and its velocity rise slows down (Fig.3).

The local transfer coefficient (6) defined by the local heat flux and the local temperature is non-linear, and is lower on the top of the substrate (Fig.4). It is because of the development of the thermal boundary layer.

$$\alpha_l(y) = \frac{\lambda_a \frac{\partial T(0, y)}{\partial x}}{T(0, y) - T_a} \tag{6}$$

The results of the 2-D model has been compared with 1-D numerical and analytical solutions for finite and infinite vertical plates, especially with results of the heat transfer governed by $\frac{1}{4}$ Law, (Fig.5) There are many definitions of convective heat transfer coefficients. For this work we introduce:

$$\alpha_{max} = \frac{P_z}{2h_s(T_{max} - T_a)}, \quad \alpha_{mean} = \overline{\alpha_l(y)} \tag{7}$$

By using the least square method one can fit the exponential functions to 2-D model results for the maximum and mean temperature, and the transfer coefficient. For such a fitting we assume that $\Delta T_0=20^\circ\text{C}$.

The results are as follows:

$$\begin{aligned} P_z &= P_0 \left(\frac{\Delta T_{max}}{\Delta T_0} \right)^n && \text{where: } P_0=5.94, n=1.12 \\ P_z &= P_0 \left(\frac{\Delta T_{mean}}{\Delta T_0} \right)^n && \text{where: } P_0=9.38, n=1.09 \\ \alpha_{max} &= \alpha_0 \left(\frac{\Delta T_{max}}{\Delta T_0} \right)^n && \text{where: } \alpha_0=5.06, n=0.11 \\ \alpha_{mean} &= \alpha_0 \left(\frac{\Delta T_{mean}}{\Delta T_0} \right)^n && \text{where: } \alpha_0=5.85, n=0.16 \end{aligned} \tag{8}$$

One of the main conclusion of this paper presented in (8) is that the transfer coefficient does not rise so fast with temperature as it has been assumed by previous works ($\frac{1}{4}$ Law). The simulations results with $n=0.11 \dots 0.16$, for $\alpha \sim T^n$. This is because of the non uniform temperature distribution on the substrate obtained by more accurate 2-D calculations. It also results in a more linear relation $P_z=f(T_{max})$, (Fig.5).

4. Experimental results

We applied thermography to confirm the computational results. The resistor was placed horizontally on the ceramic substrate. The ceramic itself was vertically situated what makes the

experiment very close to the modelling. All material parameters and dimensions were properly chosen to match the measurement and the modelling. The full confirmation is not possible, because the radiation is neglected in the model. However, the similar temperature distribution in the substrate has been found. The temperature is not symmetrical (Figs. 6-7). Due to convection, the temperature of the upper side of the resistor becomes higher than at the bottom. This result proves the necessity of including the convection in accurate modelling of heat dissipation in microelectronics. The lower maximum temperature in the measurement is caused by radiation. For thermography surveys the hybrid component has been covered with a high emissive thin layer, which increases the amount of energy emitted by radiation. The radiation will be included in the model in the near future.

5. Conclusions

This paper presents the 2-D modeling of heat dissipation by natural convection for non-uniform temperature distribution on the substrate. Although only vertically placed hybrid devices have been considered, the found results are useful wherever the temperature is non-uniform. First of all, the convective heat transfer coefficient does not vary so much with the temperature as it was predicted by (*1/4 Law*). This denotes that the maximum temperature on the substrate is almost linear versus power. 2-D simulations confirm the real need to include the convection into the accurate heat transfer modelling in microelectronics. Quite satisfactory agreement has been found between the modelling and the thermography experiment, although radiation effects should be included in the next works.

6. Acknowledgements

Author thanks Prof. G. De Mey from the University of Gent (ELIS), Belgium for his helpful scientific suggestions and inspirations which have been used in this work.

REFERENCES

- [1] Wiśniewski (S.).- Wymiana ciepła. Warsaw, WNT, 1988 [in Polish].
- [2] Holman (J.). - *Heat Transfer*, McGraw-Hill International Book Company, 1981, ISBN 0-07-Y66336-X
- [3] Rogers (D.). - *Laminar Flow Analysis*. Cambridge University Press WNT, New York 1992, ISBN 0-521-41152-1.
- [4] De Mey (G.), Boone (E.), Nachtergaele (G.), Demolder (S.) and Rottiers (L.) - Non-linear thermal Effects in Hybrid Circuits, *Journal on Hybrid Circuits*, 1990, vol. pp. 23-26.
- [5] Więcek (B.), De Mey (G.). *Evaluation of of heat dissipation by convection for VLSI circuits*. Proc. TERMINIC'96 Conf., Budapest, Sept.25-27,1996
- [6] Więcek (B.). *Radiative and convective heat transfer in microelectronics*. Proc. QIRT'96. Stuttgart, Sept. 2-5, 1996

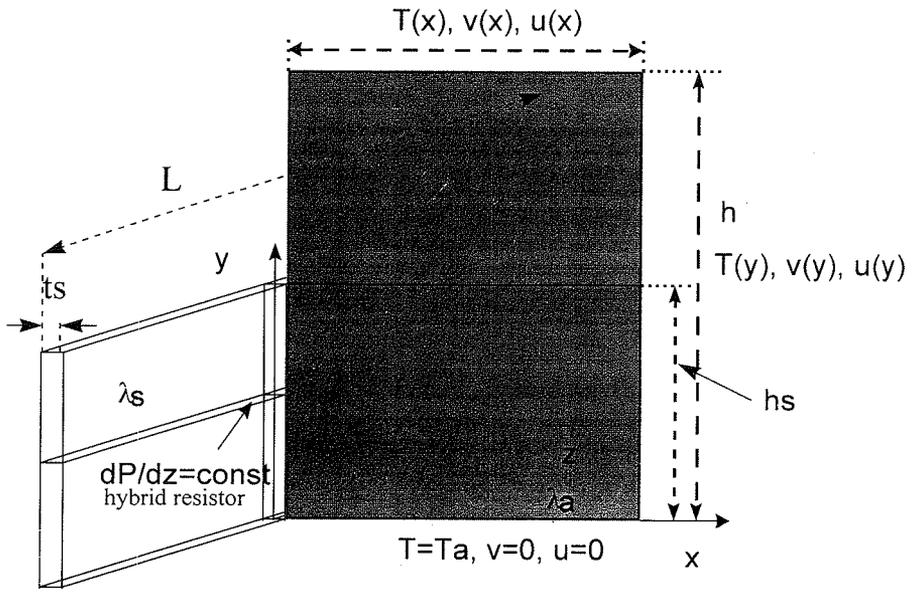


Fig.1. Area for 2-D natural convection model

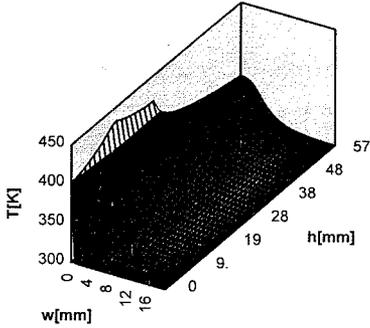


Fig.2. 2-D Temperature distribution, $P_z=50W/m$

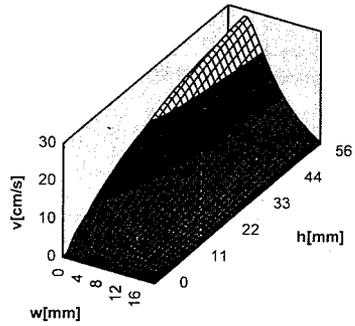


Fig. 3. 2-D vertical velocity distribution, $P_z=50W/m$

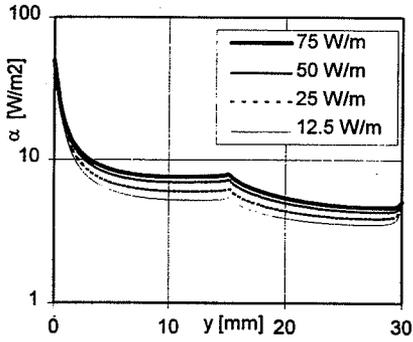


Fig. 4. Local transfer coefficient distribution

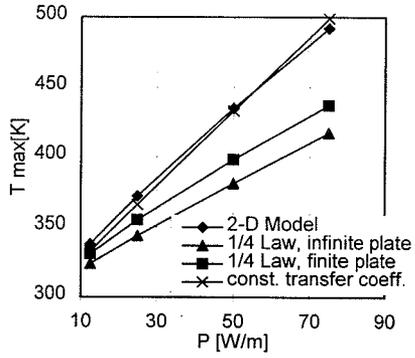


Fig. 5. T_{max} versus power

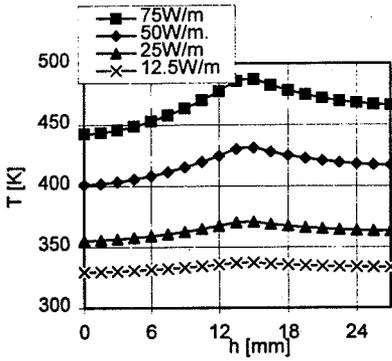


Fig.6. Non symmetrical temperature distribution on the substrate.

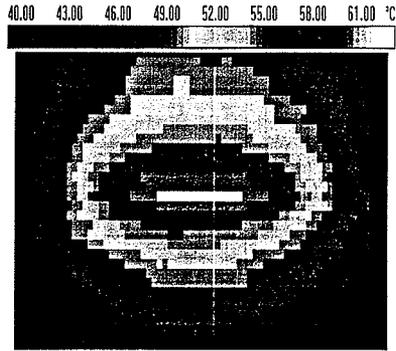


Fig.7. Non symmetrical temperature distribution on the substrate obtained by thermography measurement