Infrared thermography for local Nusselt number estimation of an elliptical fin with a transient method

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Abstract

In this paper, a transient method involving an infrared set-up is used to investigate local heat transfer over the fin of the second row of a staggered elliptical finned tube heat exchanger assembly. An experimental test bench was designed to record data during the fin cooling. The procedure of data preprocessing including IR camera calibration is presented. It allows extracting the temperature field evolution of the fin. These temperatures feed a numerical model that works with thermally thin material and takes into account lateral heat conduction and radiation with the surrounding. The heat transfer coefficient field is determined by integrating the model over time intervals that depend on space. Distributions of Nusselt number over the fin and their uncertainties are presented for several Reynolds numbers. The high resolution of the whole method and set-up allows detecting thermal imprints of developing horseshoe vortices.

1. Introduction

Finned-tube heat exchangers are used in a wide range of engineering applications, such as for automotive engine cooling, energy recovery from power plants or food processing systems. Their optimization is a key point for energy savings and costs decreasing. Increasing thermal efficiency of such heat exchangers is often achieved through the enhancement of their global air side heat transfer. In finned tubes configuration, the formation of horseshoe vortices (HSV) is observed at each fin-tube junction [1,2]. These vortical flow structures lead to intensive heat transfer rates whereas recirculation zones downstream the tubes are less efficient. Thus, estimating accurately the local heat transfer coefficient (HTC) distribution is a fundamental issue since it gives worthy information on the better way to intensify heat exchange, for instance by optimizing fins geometry and arrangements [3,4] and/or by choosing vortex generators positions [5,6].

High precision local heat transfer investigation can be achieved with transient techniques. The latters have been used with simplified heat conduction models in order to find analytical expressions of HTC. For instance, in the lumped capacitance model, the material is considered being thermally thin and lateral heat conduction into the material is neglected. With these assumptions, the HTC is expressed with natural logarithm function [7]. Another model leading to analytical expression is the one dimensional half space model [8]. The material is then supposed semi-infinite and once again lateral heat conduction is neglected. With these two models, only the elapsed time needed for the material to move from an initial temperature to a final temperature has to be evaluated. For that, liquid crystal thermography has been greatly employed [9,10]. However, it was found in [11] that neglecting the influence of lateral conduction can be erroneous even with low conductive material. Other models with less restrictive hypothesis have been used by other authors. For instance, in [12] the material is supposed to be thermally thin but lateral conduction is not neglected and radiation exchange with surrounding is taken into account. Finally, no matter what model is used, the estimation of local HTC with transient techniques consist in measuring (or calculating) the temperature field of the material surface during a thermal transient state and a steady state fluid flow. In [13,14] the authors show that the resulting HTC is quasi constant.

The aim of this study is to estimate the Nusselt number fields over the fin of the second row of a staggered elliptical finned tube heat exchanger assembly by using a transient method involving an infrared set-up. The experimental test bench is designed to record data during the fin cooling (transient thermal state but stationary aerodynamical state). Here, the procedure for data preprocessing including IR camera calibration is presented in detail. It allows extracting the temperature field evolution of the fin. These temperatures feed a numerical model that calculates the heat flux balance in the material. It is adapted for thermally thin materials and it takes into account lateral heat conduction and radiation with the surrounding. The heat transfer coefficient field is determined by integrating the model over time intervals that are depending on space and are selected with a criterion based on local transient heat transfer. The uncertainties associated to the calculation method are performed using Monte Carlo simulations. Results are shown and analysed for several Reynolds Numbers.
2. Experimental test bench

2.1. Experimental facility description

The experimental facility is sketched in figure 1. It allows investigating heat transfer characteristics of geometries placed in the test section of a subsonic blow-down open circuit wind tunnel. The test section is a rectangular shaped duct. A convergent of 10:1 area reduction factor, designed with the correlation given in [15], is connected to the duct entrance. The air is extracted from a quiescent isothermal room through honeycomb airflow straightener. These precautions allow providing uniform velocity profile at the duct inlet. Laser Doppler anemometry measurements have been performed to check the uniformity. The air flow is achieved by a frequency controlled blower, placed at the end of the line, just after a flowmeter. Finally two synchronized electrical valves (V1 and V2) and a bypass line are installed just after the test section in order to bypass it. They enable to instantaneously establish the air flow in the test section, as it will be explained in section 2.3 which presents the experimental procedure.

This study concerns the scaled up second row airside elementary pattern of a staggered elliptical finned tube heat exchanger (see figure 1). PVC elliptical tubes of diameters $1.41D_t$ and $0.71D_t$ (with $D_t$ the diameter of the equivalent round tube having the same cross section) are equidistant separated within a tube pitch of $P_t = 3.40D_t$ that is the width of the test section. PMMA elliptical fins of diameters $2.99D_t$ and $2.28D_t$ and of thickness $e_f = 1.48 \times 10^{-2} D_t$ are attached to tubes. The distance between the fin and the upper wall $P_s$ is equal to $0.272D_t$.

![Fig. 1. Experimental test bench sketch.](image)

2.2. Measurements

2.2.1. Fin temperature measurement

The HTC distribution on the fin surface is deduced from the fin temperature field evolution. The measurements are carried out by IR thermography. This is a very powerful and convenient way for local heat transfer investigation [16]. It allows accurate non intrusive measurements with high space/time resolution. The IR set-up involves a Cedip Infrared® camera (Titanium 550M: InSb detector; 320x256 pixels; 3.6-5.1μm). The fin temperature fields can be recorded thanks to the Fluorine (CaF2) IR transparent window (94% direct transmission factor in the wavelength range) that is installed on the top of the test section. Moreover the test section is equipped with a curtain system to protect the thermal scene from surrounding radiation and to avoid parasitic reflections.

The IR camera is calibrated in situ (see figure 2a). A special high conductivity plate (copper) coated with the same high emissivity coating (about 0.97) as the fin, is placed in the test section. This calibration plate has inner water circulation that is accurately thermo-regulated (Julabo temperature regulator, 0.01 °C of stability). The calibration is made
over the camera measurement range in order to extract a three order polynomial function that links the object signal OS (in digital Level DL) recorded by the IR camera to the uniform temperature ($T$ in °C) of the calibration plate (given by a probe temperature), see figure 2b. IR measurements are all made at a sampling frequency of 100 Hz.

![Diagram of calibration elements and function](image)

**Fig. 2. In situ calibration**

### 2.2.2. Other measurements

The temperature of the quiescent room $T_e$ and the temperature of surrounding walls $T_{rad}$ (i.e. the protecting curtains temperature) are also needed for the HTC calculation. They are measured with type K thermocouples that have been previously calibrated conforming to norm FD X07-029.

Flow rate in the channel flow is determined by using a diaphragm with respect to ISO 5167 norm. The leak flow has been estimated to be less than 1/1000 in the measurement range.

### 2.3. Experimental procedure

The experimental process is divided in two steps. In the first step the fan is on and the valve V1 is closed while the valve V2 is opened implying no airflow through the test section, see figure 1. A specific infrared emitting heater located above the test section warms up the fin through the infrared transparency window (figure 3a). In the second step, the infrared emitter is manually removed, and the valve V1 is opened while the valve V2 is closed steadying almost instantaneously airflow in the test section (figure 3b). During the cooling period, the infrared camera records the signal of the fin temperature drop.

![Diagram of experimental procedure steps](image)

**Fig. 3. Steps of the experimental procedure**
2.4. Measurements treatment

The collected data are firstly converted into Matlab® format and gathered in a matrix. Figure 4a is an example of recorded data at a given time. It represents the object signal in digital levels (DL) at different positions of space (in pixels). After that, all pictures are cutting-out in order to save only the data corresponding to the elliptical fin zone. For that purpose, the Laplacian of the previous figure is performed. It allows binding accurately the ellipses contours in order that the experimentator selects ten points on the ellipse that bind the tube and ten points on the ellipse that bind the fin, see figure 4b. After that the selected point’s coordinates are processed in an optimization algorithm (Levenberg-Marquardt) to find the center and both diameters of each concentric ellipse. With this information the matrix of data can be reshaped. The Laplacian is also used to delete erroneous data due to defects of camera detectors or specks of dust that may have fallen on the IR transparent window: its standard deviation is calculated and the pixels (and their neighbours) for which the Laplacian values are greater than 2.5 times the standard deviation are erased and recalculated by interpolation. Then, the calibrating function is used to convert the digital levels into temperatures (see figure 4c). Once these steps are done, data are used to calculate the HTC over the fin.

![Recorded data in DL: 320 x 256 pixels](image1)
![Contours detection from Laplacian: 173 x 229 pixels](image2)
![Fin temperature field in K](image3)

Fig. 4. Measurements treatment.

3. HTC calculation

3.1. Mathematical formulation

Let’s consider the transient heat transfer problem involving a diffusive solid medium \( \Omega \) of boundary \( \partial \Omega \) with vector \( \mathbf{n} \) as outer-pointing surface normal and \( M \) denoting a point in \( \Omega \). In this problem, the temperature is governed by equation (1) where \( \nabla^2 \) is the Laplacian operator.

\[
\forall M \in \Omega \quad \rho c_p \frac{\partial T(M,t)}{\partial t} = k \nabla^2 T(M,t)
\]  

(1)

To find the temperature in the solid, the equation has to be closed by initial condition (2) and boundary condition (3). \( T_i \) is the initial temperature. \( \nabla T \) is the temperature gradient. \( \varphi_{\text{cond}}, \varphi_{\text{conv}} \) and \( \varphi_{\text{rad}} \) are the heat fluxes exchanged by conduction, convection and radiation on boundaries.

\[
\forall M \in \Omega \quad T(M,t=0) = T_i(M)
\]  

(2)

\[
\forall M \in \partial \Omega \quad k \nabla T(M,t) \cdot \mathbf{n} = \varphi_{\text{cond}}(M,t) + \varphi_{\text{conv}}(M,t) + \varphi_{\text{rad}}(M,t)
\]  

(3)

The convection heat flux exchanged at the solid/liquid interface depends on both temperature distributions in the solid and in the fluid. Since the fluid temperature is generally hard to know, HTC \( h(M,t) \) is defined to link the heat flux and the temperature gap between a reference \( T_{\text{ref}} \) and the solid temperature \( T \). In channel flow configurations \( T_{\text{ref}} \) is often chosen as the inlet temperature or the bulk temperature. A very common way to express \( \varphi_{\text{conv}} \) is then:
\[ \phi_{\text{conv}}(M, t) = h(M, t) \left[ T_{\text{ref}} - T(M, t) \right] \]  

(4)

As testified in [13,14], the HTC resulting of a suddenly constant flow is very fast to reach a constant value. It justifies the assumption made by numerous authors of a constant HTC \( h(M) \). The next section presents a way to calculate the HTC considering this hypothesis.

### 3.2. Numerical model and solution

Any discretization method on a giving mesh of size \( Nn \) leads to a matrix equation of the form (5).

\[
\begin{align*}
\mathbf{M} \dot{T} &= \mathbf{K} T + \mathbf{F} \phi_{\text{conv}} + \mathbf{F} \phi_{\text{rad}} \\
- \mathbf{M} &\rightarrow \text{dim}(Nn \times Nn) \text{ is the mass matrix (diagonal matrix)} \\
- \mathbf{K} &\rightarrow \text{dim}(Nn \times Nn) \text{ is the stiffness matrix} \\
- \mathbf{F} &\rightarrow \text{dim}(Nn \times Ns) \text{ is a command matrix with } Ns \text{ the size of vector } \phi \text{ that contains the discretized } \phi \text{ on } \partial \Omega .
\end{align*}
\]

Since \( h \) is in \( \phi_{\text{conv}} \) (see Eq. (4)), HTC estimation problem is to determine the convection heat density \( \phi_{\text{conv}} \).

Generally, the heat densities estimation is an ill-posed inverse heat conduction problem that can be solved by using optimization method. But here the case is quite simple since the solid is assumed to be thermally thin with uniform temperature along the thickness. This last assumption is correct if the Biot number is less than 0.1. If all plate temperatures can be measured (that is the case here) then \( Ns = Nn \).

The discretized \( \phi_{\text{conv}} \) at any time is written from equation (4) by doing the element-wise product between vector \( h \) and vector \( (T_{\text{ref}} - T) \) with \( T_{\text{ref}} \) being a vector containing \( T_{\text{ref}} \) everywhere.

\[
\phi_{\text{conv}} = \text{diag}((T_{\text{ref}} - T))h
\]

The discretized \( \phi_{\text{rad}} \) at any time is written as follows (Eq. (7)), with \( \varepsilon \) the emissivity, \( \sigma \) the Stefan-Boltzmann constant and \( \bar{T}_{\text{rad}} \) the vector containing \( T_{\text{rad}} \) everywhere.

\[
\phi_{\text{rad}} = \varepsilon \sigma [\bar{T}_{\text{rad}} - T^4]
\]

Assuming \( h \) is constant, the integration of equation (5) over the time interval \([t_i, t_f]\) leads to:

\[
\mathbf{E}_{\text{tot}} = \mathbf{E}_{\text{cond}} + \mathbf{E}_{\text{conv}} + \mathbf{E}_{\text{rad}}
\]

with

\[
\mathbf{E}_{\text{tot}} = \int_{t_i}^{t_f} \mathbf{M} \dot{T} dt , \quad \mathbf{E}_{\text{cond}} = \int_{t_i}^{t_f} \mathbf{K} T dt , \quad \mathbf{E}_{\text{conv}} = \int_{t_i}^{t_f} \mathbf{F} \text{diag}((T_{\text{ref}} - T)) dt \text{ and } \mathbf{E}_{\text{rad}} = \int_{t_i}^{t_f} \mathbf{F} \phi_{\text{rad}} dt
\]

Then the HTC \( h \) can be extracted by numerical integration: see equation (10), where the bound \( i \) corresponds to time \( t_i \) and \( f \) to \( t_f \).

\[
\begin{align*}
\mathbf{h} &= \left[ \sum_{i}^{f} \text{diag}((T_{\text{ref}} - T)) \right]^{-1} \mathbf{E}^{-1} \left[ \mathbf{M} \sum_{i}^{f} T - \mathbf{K} \sum_{i}^{f} T - \mathbf{F} \sum_{i}^{f} \phi_{\text{rad}} \right]
\end{align*}
\]

(10)

It is worth to note that in [12] the author took the same integration bound \( f \) for all positions (i.e. \( \forall M \in \partial \Omega \)). This choice being quite arbitrary, it is proposed here to settle the final integration bound by using a criterion based on physical consideration: the model is integrated till a previously fixed energy lost by convection is reached. This amount of energy is chosen as a percentage \( \alpha \) of the maximal energy that the solid can lose (when flow cools the fin). This maximal energy is the initial internal energy of the material: \( \mathbf{E}_{i} \). It is calculated with equation (11) where \( \bar{T}_{i} \) is the initial temperature and \( T_{i} \) is the inlet temperature (vector containing \( T_{e} \) everywhere).
\[
E_i = \rho C_p [T_i - T_w]
\]  

(11)

Thus the integration stops when \( E_{\text{conv}} \approx \alpha E_i \). Since this criterion is local, the resulting final integration bound depends on the position.

In [17] the method has been validated on the reference case consisting in a thin plate in an aerodynamically and thermally developing channel flow. Comparison of results obtained with various values of \( \alpha \) have shown that a percentage of \( \alpha = 15\% \) is a good choice as it leads to low differences between experimental results and reference results (Stephan correlation), and the final integration times \( t_f \) is relatively weak (computing resources saving).

3.3. Developed algorithm

The algorithm of figure 5 has been implemented in Matlab®. The spatial discretization is based on finite volume method and the time discretization is fully implicit. The seeking of integration bounds can be made in different way. In this study the dichotomy method was chosen.

4. Results

Experiments on the elliptical fin are performed with configuration of figure 1 for several Reynolds numbers based on the round tube equivalent diameter \( D_t \): \( Re_{D_t} \) from 1300 to 7800. The spatial resolution is such that the thermal scene size, which fits the whole elliptical fin, is 173x229 square pixels of \( 1.31 \times 10^{-2} D_t \) side. Estimated Nusselt number fields are presented and analyzed. They are calculated with \( Nu_{D_t} = h D_t k_{\text{air}} \).

4.1. Topology description

Dynamical and thermal phenomena occurring around an elliptical finned tube are comparable to the ones around a circular finned tube: when a circular finned tube is placed in a uniformly distributed flow, that is parallel to the fin, horseshoe vortices (HSV) appear at the fin-tube junction. Figure 6 presents typical streamlines upstream a circular finned tube (only half of the fin spacing \( Ps/2 \) is represented). These streamlines show the boundary layer development from the fin leading edge and the flow separation which occurs upstream the tube. In the separation zone a complex 3D HSV structure develops. In figure 6, it is composed of two primary vortices (\( P_1 \) and \( P_2 \)) and one secondary counter-rotating vortex (\( S_1 \)) localized between both primary vortices. These vortical cells developing upstream wrap then around the tube to form a complex three-dimensional U-shaped structure. The number of HSV as well as their positions and sizes vary with parameters like Reynolds number and fin spacing [18,19]. HSV have been intensively studied during the last years. Indeed measurement techniques like particle image velocimetry (PIV) [20], and infrared (IR) thermography [12] as well as computational fluid dynamics (CFD) capabilities [21] allow investigating more and more precisely the development mechanisms of such flow structures. Nevertheless, in literature, it is very difficult to find experimental results presenting local HTC of finned tube with sufficient resolution to detect accurately HSV thermal imprints. However, these data would be very useful to validate numerical simulations.
4.2. Nusselt Number distribution over the elliptical fin

Figure 7 gives the local Nusselt number fields for two of the three considered Reynolds numbers. The great precision of the method and the high spatial resolution enable to reveal distinctly symmetrical U-shaped distributions around the tube. These imprints are the consequence of primary HSV. Some previous experimental results [22] have shown that these thermal imprints are linked to primary vortices locations: the vorticity of secondary horseshoe vortices is so low that their effect is not detectable on $\bar{N_u}_D$ fields. Comparing imprints of both $\bar{N_u}_D$ fields underlines that several vortices appear as the Reynolds number rises. For $\text{Re}_{Dt}$ of 1294, figure 7a presents one primary vortex thermal imprint ($P_1$). Rising $\text{Re}_{Dt}$ to 2588 (figure 7b) leads to the apparition of a second primary HSV thermal imprint ($P_2$). The $\bar{N_u}_D$ levels also reveal that the fin leading edge and the HSV system highly contribute to heat exchange. On the contrary the recirculation region downstream the tube is less efficient. It is due to stagnation flow in this dead zone. In figure 8 the Nusselt numbers along the line 1 upstream the fin / tube junction (line 1 is defined in figure 7a) are plotted for four different Reynolds numbers. Upstream the tube (on line 1) $\bar{N_u}_D$ follows a decrease from the leading edge ($z/Dt$ ranging from 1.41 to 0.98). This corresponds to the development of the boundary layer typically observed in such developing channel flows. Increasing $\text{Re}_{Dt}$ intensifies heat exchange in the leading edge zone because of the thinning of the boundary layer. It also intensifies the levels of maximal local $\bar{N_u}_D$ of HSV imprints. Furthermore a change of the thermal imprints location away from the tube / fin junction can be noticed. This translation is accompanied with a slight thickening of the imprints. This behavior is linked to the HSV growth with $\text{Re}_{Dt}$ increase as shown in [23]. Finally, Nusselt number fields $\bar{N_u}_D$ are integrated over the fin area to give the mean values $\langle \bar{N_u}_D \rangle$ which quantify the global exchange. Results of figure 9 show obviously that the global heat transfer grows as $\text{Re}_{Dt}$ rises.

![Fig. 6. Typical flow topology: horseshoe vortex formation upstream a circular finned tube (from [17]).](image)

![Fig. 7. Nusselt Number field for the elliptical fin](image)
Figure 10 gives the final integration time fields corresponding to previous $Nu_{Dt}$ distributions of figure 7. These fields are found using the criterion presented at the end of section 3.2 with the user parameter $\alpha$ equal to 15%. The integration time repartitions are found absolutely not uniform. Their variations are the inverse variations of Nusselt number because of the way the integration bounds are chosen (i.e. from a given amount of energy transferred). That is why there are also U-shapes in final time integration fields.

4.3. Uncertainties

The local HTC uncertainty linked to the calculation method is estimated using Monte Carlo approach [24]. Measured parameters uncertainties are evaluated given instruments resolution whereas thermo-physical properties uncertainties are provided by material suppliers. These values, in s.i. units, are reported in Table 1. The local HTC is assessed, with randomly chosen parameters values, as many times as needed to see the convergence of its standard deviation to 1E-3. For that, the distribution of $w$ (that represent any parameter) is supposed to follow normal law (Gaussian) with standard deviation equals to $\Delta w/3$, such as 99.7% of values are in confidence intervals $[w-\Delta w,w+\Delta w]$. Then the local HTC uncertainty is obtained by multiplying by 3 the found converged standard deviation.
Table 1. Parameters uncertainties in s.i. units

<table>
<thead>
<tr>
<th></th>
<th>w</th>
<th>ef</th>
<th>$T_e$</th>
<th>$T_{rad}$</th>
<th>$T$</th>
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<tr>
<td>$\Delta w$</td>
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<td>±0.2</td>
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<td>±0.1</td>
<td>±0.01</td>
<td>±15</td>
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Relative uncertainty fields are plotted in figure 11 for $Re_{Dt}$ of 1294 and 2588. The highest values are noticed for the smallest Nusselt number values in the recirculation region whereas they are very low in the leading edge zone and in the HSV imprints region. Finally the averaged uncertainties are no more than 4.7% for $Re_{Dt}$ of 1294 and 4.4% for $Re_{Dt}$ of 2588. These low values of Nusselt number uncertainties demonstrate the accuracy of the method.

![Air flow](image)

Fig. 11. Local relative uncertainties of Nusselt number fields

5. Conclusion

An experimental set-up involving IR thermography has been designed to estimate local heat transfer coefficient (HTC) distribution with a transient technique. The method consists in time integration of a heat conduction model that takes into account lateral heat conduction into the material and radiation with surrounding. The integration bounds depend on space. They have been chosen in function of the evolution of local heat transferred by convection. The method is applied to assess the local HTC over the fin of the second row of a staggered elliptical finned tube heat exchanger assembly. Nusselt number is calculated for several Reynolds numbers ranging from 1300 to 7800. The thermal imprints of HSV are precisely detected thanks to the high resolution of IR set-up and accuracy of the developed method. The presented results corroborate previous studies about HSV behaviors. The locations of HSV thermal imprints as well as the leading edge zones are regions of intensive heat exchange whereas the recirculation zone downstream the tubes is less efficient. Such local HTC estimation constitutes a fundamental step towards passive intensification of heat transfer in heat exchangers as it provides key elements for a wise choice of fins geometry and/or complementary vortex generators locations. Moreover these experimental results are of great interest for validating numerical simulations.

REFERENCES


