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# Finite element model for crack characterization by lock-in thermography

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**Abstract**

In this work, a discontinuous Galerkin finite element method, for a discontinuous transmission problem, is proposed to model the thermal wave scattering in lock-in thermographic inspection of cracked opaque samples with general inner cracks. Unlike continuous finite element methods, the model we present does not require meshing of the gaps within the cracks, radically reducing the amount of degrees of freedom and the elapsed time spent during the simulation. We will focus on analyzing the sensitivity of the temperature propagation through the crack under different parameters such as length, depth, thermal resistance of finite cracks and parameters involving the modulated laser excitation.

## 1. Introduction

Photothermal techniques have great capacity for subsurface crack detection [1]. The application of computational techniques, such as finite differences for pulsed or modulated thermography [2], or continuous finite element method (FEM) for lock-in thermography [3], provides a complementary tool for crack characterization and detection. Continuous FEM requires a fine meshing of the cracked domain, dramatically increasing the number of degrees of freedom and therefore the computational time.

In this work, a Bauman-Oden-type discontinuous Galerkin (DG) formulation [4] is used to solve a discontinuous transmission problem modelling the thermal wave scattering in lock-in thermographic inspection of cracked opaque samples, with general inner cracks characterized by their thermal resistance [5]. The DG-model we present reduces the amount of degrees of freedom and the elapsed time spent during the simulation, since it does not require meshing of the gaps within the cracks.

The 3D heat flow simulation model has been implemented by using a collection of scientific open-source software, NETGEN as mesh generator, FEniCS [6] for automated solution of differential equations by FEM and ParaView for data visualization.

We will focus on studying the sensitivity of the temperature propagation through the cracks, under different parameters such as length, depth, thermal resistance of finite cracks and parameters involving the laser excitation.

## 2. Modelization

Let be considered a cracked domain $\Omega$, depicted in figure 1, with thermal conductivity $\kappa$ and thermal diffusivity $\alpha$; the crack, located on the interface $\Gamma_c$, is characterized by a thermal resistance $R$. An oscillating heat flux of amplitude $g$ at frequency $f$ is applied on the illuminated boundary $\Gamma_g$, the remaining boundary $\Gamma_0$ is under adiabatic conditions. The spatial component $u(\boldsymbol{x})$, of the thermal wave $\Re\left(u(\boldsymbol{x})e^{-i2\pi f t}\right)$ induced into $\Omega$, is governed by the discontinuous transmission problem shown in Eq.(1).

$$\begin{cases} \Delta u(\boldsymbol{x}) + i\frac{2\pi f}{\alpha} u(\boldsymbol{x}) = 0\,, & \boldsymbol{x} \in \Omega \backslash \Gamma_c, \\ \left(\kappa \frac{\partial u_-}{\partial \boldsymbol{n}_-} + \kappa \frac{\partial u_+}{\partial \boldsymbol{n}_+}\right)\Big|_{\Gamma_c}(\boldsymbol{x}) = 0, & \boldsymbol{x} \in \Gamma_c, \\ (u_- - u_+)|_{\Gamma_c}(\boldsymbol{x}) + R(\boldsymbol{x})\kappa \frac{\partial u_-}{\partial \boldsymbol{n}_-}\Big|_{\Gamma_c}(\boldsymbol{x}) = 0, & \boldsymbol{x} \in \Gamma_c, \\ \frac{\partial u}{\partial \boldsymbol{n}}\Big|_{\Gamma_0}(\boldsymbol{x}) = 0, & \boldsymbol{x} \in \Gamma_0, \\ \kappa \frac{\partial u}{\partial \boldsymbol{n}}\Big|_{\Gamma_g}(\boldsymbol{x}) = g(\boldsymbol{x}), & \boldsymbol{x} \in \Gamma_g. \end{cases} \tag{1}$$

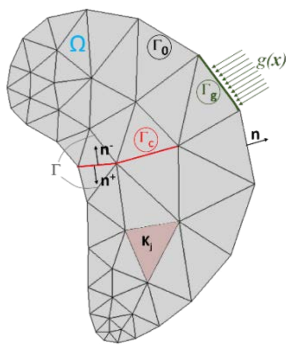


**Fig. 1.** *Meshed domain comforms with the crack.*

The subscripts + and – denote the right and left sides from the point of view of an observer sited on the crack (or any facet of the triangulation), the outward normal vectors to each side are denoted as $\boldsymbol{n}_+$ and $\boldsymbol{n}_-$respectively.

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Four steps must be performed to solve the problem exposed in Eq. (1) by DG finite element,:

1. To define a mesh of elements $K_j$, $j = 1, \dots M,$ usually triangles for 2D and tetrahedra for 3D.
2. To choose a kind of simple piecewise functions $u_h$ to approximate the temperature $u$ on each element.
3. To introduce the *averages* $\{\cdot\}$ and *jumps* $[[\cdot]]$ traces, on the skeleton $\Gamma$ of the mesh and the interface $\Gamma_c$, for scalar or vector valued functions on $\Omega$, defined as

$$\{\cdot\} = \frac{1}{2}((\cdot)^+ + (\cdot)^-) \qquad \text{and} \qquad [\![\cdot]\!] = (\cdot)^+ \cdot \boldsymbol{n}_+ + (\cdot)^- \cdot \boldsymbol{n}_- \tag{2}$$

4. To deduce an stable DG variational formulation of Eq.(1).

Specifically, we choose polynomial (of degree greater than or equal to 2) piecewise functions and a Bauman-Oden-type DG variational formulation, Eq. (3), characterized by the additional stabilizer term located in the third addendum on the left hand side of Eq. (3),

$$\sum_{j=1}^{M} \int_{K_j} \nabla u_h \nabla v_h - \int_{\Gamma} \{\nabla u_h\}[\![v_h]\!] + \int_{\Gamma} [\![u_h]\!]\{\nabla v_h\} + \frac{1}{k}\int_{\Gamma_c} \frac{1}{R}[\![u_h]\!][\![v_h]\!] - i\frac{2\pi f}{\alpha}\int_{\Omega} u_h\, v_h = \int_{\Gamma_g} g\, v_h \tag{3}$$

## 3. DG analysis of lock-in thermograms

The presented DG formulation allows to obtain results for any sample geometry with cracks of any size, shape or thickness. As an applied example, figure 2 shows the numerical thermograms obtained on the illuminated surface of a transversally cracked prism with AISI-304 stainless steel thermal properties. This resembles a transversal air-crack of 25 µm. A Gaussian laser spot of radius 0.5 mm illuminates both sides of the crack in a non-symmetric way.

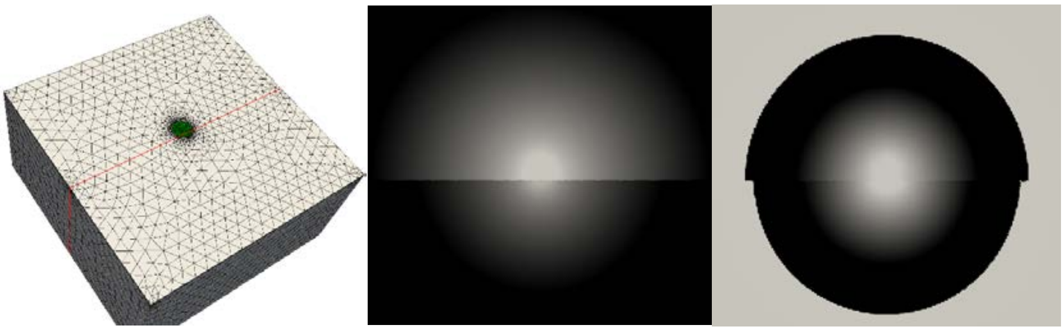

**Fig. 2.** *(Left) Block meshing of an AISI-304 cracked stainless steel, green disk shows the position of the 0.8 Hz modulated laser beam of radius 0.5 mm, red line represents a crack with $R = 10^{-3}$ $m^2KW^{-1}$. The center of the laser spot and the crack are separated 0.3 mm. (Center) Numerical amplitude thermogram. (Right) Numerical phase thermogram.*

## 4. Current and future work

We are working in coordination with the Photothermal Techniques Laboratory UPV/EHU in Bilbao in order to effectively being able to detect and characterize cracks from thermography data in industrial materials.

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